

B.Sc. Semester - 1 (NEP-2020) Examination
OCT/NOV-2025 (As Per Syllabus Year June-2023)

MAT: Matrices and Coordinate Geometry (Multidisciplinary Course)

Time: 2:00 Hours

Marks:50

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any one question out of two. (05)

- (1) in usual notations, prove that the multiplication of matrices is associative.
- (2) Define: Period of matrix, index of matrix, involutory matrix, unitary matrix, Skew symmetric matrix.

Que.1(B) Answer any one question out of two. (05)

- (1) Prove that every square matrix can be expressed uniquely as the sum of a symmetric and a skew symmetric matrix.
- (2) in usual notations prove that (i) $(AB)^{-1} = B^{-1}A^{-1}$ (ii) $(A^T)^{-1} = (A^{-1})^T$

Que.2(A) Answer any one question out of two. (05)

- (1) Prove that the row rank of a matrix is the same as its rank.
- (2) Find rank of matrix $\begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$ using linearly independent rows or columns.

Que.2(B) Answer any one question out of two. (05)

- (1) Define rank of matrix and Show that rank of any matrix $\geq$ rank of its every sub matrix.

- (2) Define echelon form of a matrix and find rank of matrix $\begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$

Using echelon form.

Que.3(A) Answer any one question out of two. (05)

- (1) in usual notations, Prove that the equation $AX = O$ has a non zero solution iff A is singular.

- (2) Find inverse of matrix $\begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix}$ using Cayley-Hamilton theorem.

Que.3(B) Answer any one question out of two. (05)

- (1) Show that there is unique eigen value with respect to any eigen vector of given matrix

- (2) Find value of a and b such that the equations ;

$$x-2y+3z=1$$

$$3x+y-z=4$$

$$2x-2y+az=b \text{ have an infinite number of solutions.}$$

Que.4(A) Answer any one question out of two. (05)

- (1) if P'P and Q'Q are the perpendicular focus chords of a conic then prove that $\frac{1}{SP \cdot SP'} + \frac{1}{SQ \cdot SQ'} = \text{constant}$.

- (2) Derive relation between Cartesian coordinates and polar coordinates.

Que.4(B) Answer any one question out of two. (05)

- (1) Derive the pedal equation of a conic $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4} \left(\frac{dr}{d\theta}\right)^2$
- (2) Find the polar equation of the circle whose diameter has extrimities (2,0) and $(3, \frac{\pi}{2})$.

Que.5(A) Answer any one question out of two. (05)

- (1) If A is a square matrix of order n, then prove that $A(\text{adj}A) = \det(A) I_n$.
- (2) Find Eigen values and Eigen vectors of matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

Que.5(B) Answer any one question out of two. (05)

- (1) Solve following linear equations :
 $2x - y + 3z = 9$
 $x + y + z = 6$
 $x - y + z = 2$
- (2) Find area of a triangle having vertices $(-3, -30^\circ)$, $(5, 150^\circ)$, $(7, 210^\circ)$.
